New Trends in Mathematical Physics at the Interface of Analysis and Probability - List of Abstracts

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Stefan Adams

Variational problems for Laplacian interface models in $(1+1)$ *dimensions*

We obtain variational problems for the free energy of a Laplacian interface model which is a Hamiltonian system with a bi-Laplacian operator. We study scaling limits and the corresponding large deviation principles perturbed by an attractive force towards the origin to complete the microscopic-macroscopic transition. In particular we analyse the critical situation that the rate functions admit more than one minimiser leading to concentration of measure problems. The interface models are a class of linear chain models with Laplacian interaction and appear naturally in the physical literature in the context of semi-flexible polymers. We discuss these connections as well as the ones with the related gradient models. These random fields are a class of model systems arising in the studies of random interfaces, critical phenomena, random geometry, field theory, and elasticity theory. If time permits we outline open questions in higher dimensions, that is $(d+m)$ -dimensional models and their large deviation principles.

Roland Bauerschmidt

Eigenvectors and spectral measure of random regular graphs of fixed degree

I will discuss results on the delocalisation of eigenvectors and the spectral measure of random regular graphs with large but fixed degree. Our approach combines the almost deterministic structure of random regular graphs at small distances with random matrix like behaviour at large distances.

Eric Cancés

Incommensurate and disordered quantum systems

After recalling the standard mathematical formalism used to model disordered materials such as doped semiconductors, alloys, or amorphous materials, and classical results about random Schroedinger operators (Anderson localization), I will present a tight-binding model for computing the electrical conductivity of multilayer 2D materials. All these models fall into the scope of the mathematical framework, based on non-commutative geometry, introduced by Bellissard to study the electronic properties of aperiodic systems. I will finally present numerical calculations of the electronic conductivity of 1D incommensurate bilayer systems as a function of the lattice constant ratio and the Fermi level. The plot of the so-obtained function is reminiscent of Hofstadter's butterfly.

Djalil Chafaï

Concentration for Coulomb gases and Coulomb transport inequalities

This talk will present a recent joint work with Mylene Maida and Adrien Hardy on the non-asymptotic behavior of Coulomb gases in dimension two and more. Such gases are modeled by an exchangeable Boltzmann-Gibbs measure with a singular two-body interaction. We obtain concentration of measure inequalities for the empirical distribution of such gases around their equilibrium measure, with respect to bounded Lipschitz and Wasserstein distances. This implies macroscopic as well as mesoscopic convergence in such distances. In particular, we improve the concentration inequalities known for the empirical spectral distribution of Ginibre random matrices. Our approach is remarkably simple and bypasses the use of renormalized energy. It crucially relies on new inequalities between probability metrics, including Coulomb transport inequalities which can be of independent interest.

Jean-Dominique Deuschel

A local limit theorem for 2-d conductance model with application to gradient interface model

We consider a symmetric random walk in random environment and show the convergence of the corresponding rescaled 2-dimensional potential. Using the random walk representation this yields the asymptotic of the variance in the 2-d anharmonic gradient interface model.

Simone di Marino

DFT, multimarginal optimal transport and Lieb-Oxford inequalities

We first review the Density Functional Theory and its link with multimarginal optimal transportation. Then we will focus on the Lieb-Oxford inequality which is an estimate from below of the optimal transportation cost; we will show that this inequality is exact in the limit N to infinity for the one dimensional case.

Gero Friesecke

Density functional theory and optimal transport with Coulomb cost

Augusto Gerolin

A counterexample in SCE Density Functional Theory

The Strictly-Correlated-Electrons (SCE) density functional theory (SCE DFT) approach, originally proposed by Michael Seidl, is a formulation of density functional theory, alternative to the widely used Kohn-Sham DFT, especially aimed at the study of strongly-correlated systems. Following the talk of Gero Friesecke, we will recall briefly a link between SCE DFT and Multi-marginal Optimal Transport (OT) Theory and discuss the main issues of SCE DFT through the OT framework. Finally, we will present a counterexample of the existence of a "Seild map" for a class of radially symmetric densities in $\mathbb{R}^3$.

Håkan Hedenmalm

Bloch functions, asymptotic variance, and geometric zero packing

Abrikosov's analysis from 1957 of type II superconductors involves an energy functional which when suitable localized becomes the problem of determining the minimal L^4 norm of a section given that the L^2 is fixed. The problem is essentially one of determining the location of the zeros of a wave function in the lowest Landau level with minimal L^4 average given the L^2 average. This model problem has been considered by F. Nier et al. It is believed that the equilateral triangular configuration of zeros is optimal. Here we find a relation between an analogous hyperbolic geometry problem and a sought-after constant in quasiconformal theory. We prove that a related hyperbolic density is positive which then gives that the quasiconformal constant is < 1. We also discuss the general minimization problem on compact surfaces, depending on the genus.

Roman Kotecký

Metastability for a model on continuum

Ben Leimkuhler

Stochastic differential equations and numerical methods for multimodal Gibbs sampling

Problems in molecular simulation and data analytics demand new types of sampling algorithms to efficiently traverse the landscapes of models with energetic and entropic barriers. I will compare several approaches based on modified stochastic differential equations which can provide enhanced sampling efficiency. I will also highlight the importance of numerical method design in obtaining optimal performance.

Mathieu Lewin

Mean-field limits for bosons and fermions

In this talk I will review recent works in collaboration with Soeren Fournais, Phan Thanh Nam, Nicolas Rougerie and Jan Philip Solovej on the mean-field limit for quantum systems. This is a regime in which the number of particles N tends to infinity and the interaction strength behaves as 1/N. In particular I will insist on the difference between bosons and fermions, and make some connections with results for classical gases.

Jason Miller

Convergence of the self-avoiding walk on random quadrangulations to $SLE_{8/3}$ on $\sqrt{8/3}$-Liouville quantum gravity.

Let (Q, λ) be a uniform infinite quadrangulation of the half-plane decorated by a self-avoiding walk. We prove that (Q, λ) converges in the scaling limit to a certain $\sqrt{8/3}$-Liouville quantum gravity surface decorated by an independent chordal $\mathrm{SLE}_{8/3}$. The scaling limit can equivalently be described as the metric gluing of two independent instances of the Brownian half-plane. The topology of convergence is the local Gromov-Hausdorff-Prokhorov-uniform topology, the natural generalization of the Gromov-Hausdorff topology to curve-decorated metric measure spaces. This is joint work with E. Gwynne.

Christoph Ortner

Separability and Locality of Energy for the Tight-Binding Model and some Applications

I will review some recent results on the locality of interaction in the tight-binding model (treated as a toy-model for quantum chemistry). Specifically, I will show how one can decompose the density of states into spatially localised contributions. I will show two applications of this technique: (1) a proof of equivalence of canonical and grand-canonical ensembles for the electrons; (2) construction of multi-scale methods with controlled approximation errors. (joint work with Huajie Chen and Jianfeng Lu); (3) a generalisation of Brillouin-zone sampling to incommensurate layers of 2D lattices. Time permitting I will also discuss ongoing work (with Hong Duong) on an analogous decomposition of free energy in the harmonic approximation.

Nadia Sidorova

Delocalising the parabolic Anderson model

The parabolic Anderson problem is the Cauchy problem for the heat equation on the integer lattice with random potential. It is well-known that, unlike the standard heat equation, the solution of the parabolic Anderson model exhibits strong localisation. In particular, for a wide class of iid potentials (including Pareto potentials) it is localised at just one point. In the talk, we discuss a natural modification of the parabolic Anderson model on Z, which exhibits a phase transition between localisation and delocalisation. This is a joint work with Stephen Muirhead and Richard Pymar.

Benjamin Stamm

Continuum solvation models for the modelling of electrostatic interaction between solvent and solute molecules

The large majority of chemically interesting phenomena take place in liquid phase, where the environment (e.g., solvent) can play a crucial role in determining the structure, the properties and the dynamics of the system to be studied. In a practical context, accounting for all solvent molecules explicitly mat be infeasible due to the complexity of the underlying equations. A particular choice to reduce the complexity is to model the solvent to be a polarisable continuum medium. The resulting electrostatic energy contribution to the solvation energy can be computed by solving a Poisson-type interface problem. To design a fast and efficient electrostatic solver is a delicate task as the electrostatic potential only decays slowly, i.e. with a rate 1/r, towards infinity. We refer to integral equations on the interface between the solvent and the solute in order to discretize the problem using a new domain decomposition paradigm for integral equations.

Hendrik Weber

Equilibration for the dynamical Φ^4 model

In this talk I will discuss the long term behaviour of the stochastic PDE $\partial_t \phi = \Delta \phi - \phi^3 + \xi$, where ξ denotes space-time white noise and the space variables x takes values in the d dimensional torus for either $d = 2, 3$. This equation was proposed in the eighties by Parisi and Wu to give a dynamical construction of the Euclidean Φ^4 quantum field theory which (at least formally) arises as the invariant measure of this SPDE. Due to the irregularity of the driving white noise, the constructing solutions

to the SPDE was an open problem for many years - the construction of short time solutions in the more difficult three dimensional case was accomplished by Hairer only a few years ago. In this talk I will go back to Parisi and Wu's original question and study the long term behaviour of solutions. In the two dimensional case $d = 2$ I will show that solutions converge to equilibrium exponentially fast. I will also outline the proof of a similar statement in the three dimensional case. This is based on joint work with Pavlos Tsatsoulis and Jean-Christophe Mourrat.

Johannes Zimmer

Particles and the geometry/thermodynamics of macroscopic evolution

One often aims to describe the collective behaviour of an infinite number of particles by the differential equation governing the evolution of their density. The theory of hydrodynamic limits addresses this problem. In this talk, the focus will be on linking the particles with the geometry of the macroscopic evolution. Zero-range processes will be used as guiding example. The geometry of the associated hydrodynamic limit, a nonlinear diffusion equation, will be derived. Large deviations serve as a tool of scale-bridging to describe the many-particle dynamics by partial differential equations (PDEs) revealing the geometry as well. Finally, we will discuss the near-minimum structure, studying the fluctuations around the minimum state described by the deterministic PDE.